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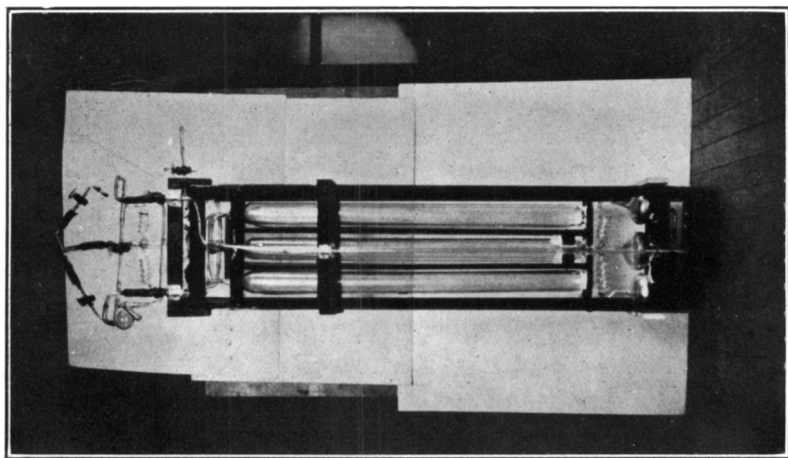
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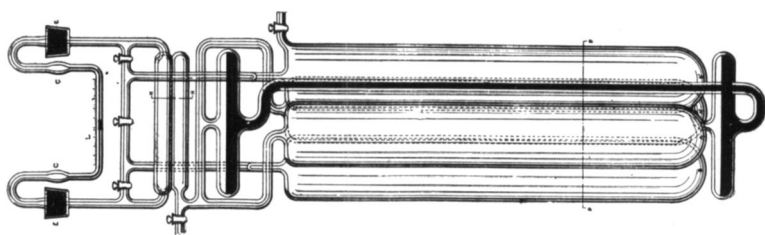
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PLATE II. A DIFFERENTIAL GRAVIMETER



A DIFFERENTIAL GRAVIMETER AND ITS APPLICATIONS¹

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The fact that mutual attraction of masses, which in the case of the Earth we call gravity, is the force which rules the material universe, is so generally known that it does not need to be emphasized here. But altho the law of attraction is the best established and the most universal one, its nature is the most mysterious and the most hidden to our research. We do not even know if attraction acts thru the medium separating the bodies, as the law of the inverse square seems to point out; or if it has an immediate effect in masses without any action thru the medium; if it is propagated thru the medium we have no knowledge whatsoever about the speed of this propagation. F. Tisserand in his wonderful "Traité de Mécanique Céleste," devotes the whole of Chapter XXVIII of Volume IV to the subject, "Vitesse de propagation de l'attraction"; but his first words are the frank statement "on ne sait rien à ce sujet"; then, on the supposition of Laplace that attraction is due to the impulsion of a fluid, he arrives at the conclusion that such an impulsion should be propagated at least 6×10^6 times faster than light; while on the other hand, according to Einstein's theory, the velocity of light cannot be surpassed by any other velocity in the physical universe. Furthermore, physics has means for isolating sound, heat, magnetism, electricity, light; we have, so to speak, control of all these forces, but we are, or at least we are supposed to be, absolutely powerless to modify the effect of gravitation; let us say, we cannot change the weight of the mass.

The mechanical revolution which would be caused by the discovery of means for modifying the field of gravity is beyond the power of our imagination to picture. Yet, is it an intrinsic impossibility? In such a case all our attempts would be useless; we cannot change the nature of things. But if the attraction of two masses acts thru the medium separating the bodies, and, if in any way (excepting, of course, the change in distance) the effect of this cause can be modified, then an instrument to detect immediately

¹An elaboration of the paper read at the 22nd meeting of the American Astronomical Society, held at Harvard Observatory, September, 1918.

any little difference in the force of attraction, cannot but have great importance in the study of the universal law.

The *pendulum* measures, indeed, with great accuracy the force of gravity, but due to its inertia *cannot detect a sudden change in this force*; it could be compared to a thermometer which could indicate only when or where *it is* cooler or hotter, but not when or where *it becomes* cooler or hotter. The instrument devised by the writer, the description of which will be the object of the present article, has above all the great advantage of responding instantaneously to any change of the gravitational field.

The principle consists in the use of a weight, mg , as a means of producing a certain pressure in a volume of gas. The variation of gravity will cause a corresponding change in the pressure, and consequently in the volume of gas, such, that the increment of volume, ΔV , will be proportional to the increment of g .

The simplest application of this principle would be to use a balloon full of gas, with a tube of constant cross-section (Fig. 1) containing a little mercury; the gas will be drawn off until its pressure is approximately equal to that of the column of mercury, the vacuum remaining in the upper part of the tube. The height and the mass of the column of mercury being constant, the pressure and therefore the volume of gas, will be a function of the gravity g .

The formula giving this relation is $\Delta v_t = v_0 \left(\alpha \Delta t - \frac{\Delta g_0}{g_0} \right)$, where v_0 is the volume at the given temperature t , for a given value of g_0 , and α is the coefficient of expansion for gases. With such an instrument, in order to secure $1 \cdot 10^{-5}$ in the variation of g , the temperature should be measured, or at least kept constant within one three-hundred sixty-seventh of a degree Centigrade; this is a difficulty but not the one which makes this arrangement unsuccessful in practice. The true and only reason why I had to give up this model after various trials, is the friction of the column of mercury which prevents it from being moved along the tube by a force equivalent to $1 \cdot 10^{-5}$ of its weight, as would be necessary in order to get results of any scientific value.

An apparatus designed by Mascart and modified by Issel under the name of *gasvolumeter* has some fundamental resemblance to the one explained above, but Issel's instrument offers still greater disadvantages, as the height of the column of mercury does not remain constant, and the friction, far from being diminished, is still

more increased. (See Svante Arrhenius, *Lehrbuch der Kosmischen Physik*, Leipzig, 1903, p. 246.) In this book the failure of the apparatus is attributed to the effect of temperature; but on bringing the instrument from the description to the laboratory it is easily seen that the real limit of its accuracy is set not by the temperature, which nowadays can be measured within a millionth of a degree, but to the friction of mercury in the capillary tube, as stated above.

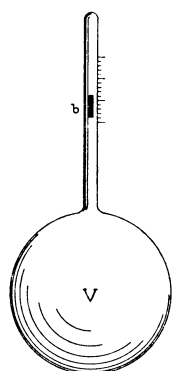


FIG. 1.

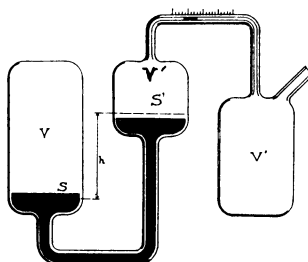


FIG. 2.

Five years having elapsed since the presentation of my first model to the Academy of Science in Barcelona, Spain, I devised another arrangement which, while retaining the fundamental principle, makes of it a completely new instrument in which friction is practically avoided. Its two essential features are as follows: first, to have a column of mercury with broad terminals, so as to make ΔV , the increment of volume, the greatest possible for any given change of its height due to the change of g ; secondly, to measure this increment of volume by dividing it into two sections separated by a capillary tube in the interior of which there is a drop of alcohol as indicated in Fig. 2.

The pressure at V is equal to the pressure at V^1 plus the weight of the column of mercury h ; as soon as the mercury changes its weight, due to the change of gravity, equilibrium will no longer subsist, and the pressure at V will be able to raise the column of mercury until this increment of height Δh , together with the corresponding increment of pressure at V^1 , due to the diminution of this volume, will re-establish the equilibrium. The formula giving the

increment of h , on the supposition of a constant and a uniform temperature, is $\Delta h = \frac{\Delta g}{g} \cdot \frac{h}{1 + \frac{(2a+h)S}{V}}$, where S is the chamber's cross-

section, a , the pressure at V^1 , and V supposed equal to V^1 .

If we can measure experimentally Δh , or the increment in the column of mercury, we can deduce the increment in the force of gravity. If the terminals of the column of mercury have a great surface, let us say 200 sq. cm. or 20,000 sq. mm, which corresponds to a circle about six and a half inches in diameter, then an incre-

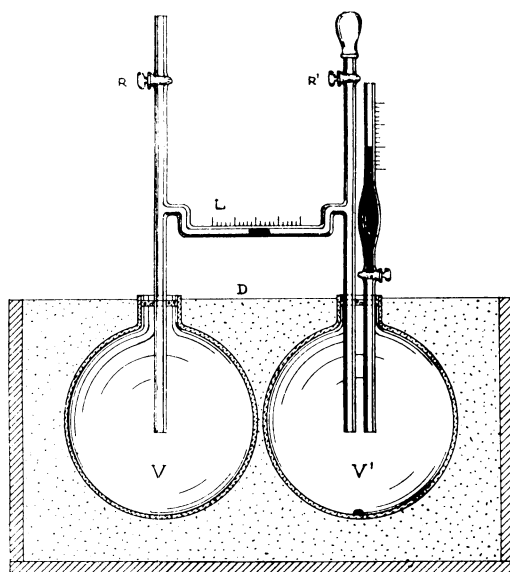


FIG. 3.

ment Δh of only one ten-thousandth of a millimeter will cause an increment of volume equal to Δh multiplied by S , or $10^{-4} \times 2 \cdot 10^4$, two cubic millimeters; but Δh is evidently equal to $\frac{\Delta V}{S}$, and hence it will be enough to measure ΔV experimentally. This increment of volume, given by the formula $\Delta V = \Delta h S = \frac{\Delta g}{g} \cdot \frac{h}{1 + \frac{2a+h}{S} \frac{S}{V}}$ will take

place on both volumes with different signs, but on the one limited by S^1 will produce a corresponding change in the pressure and push

the drop of ether or alcohol towards the right until equilibrium is re-established.

In order to have reliable results, it is necessary to establish: *First*, that the index of ether or alcohol will be moved by so small a difference in pressure; *secondly*, that we can eliminate the effect of temperature. For testing the first I arranged two balloons about one liter each in volume, as indicated in Fig. 3. It was enough to reduce in some millionths the volume of gas in V^1 by dropping a few millimeters of mercury into it, in order to have an immediate displacement of the index towards the left. By pressing the small rubber bulb the same effect was obtained, and as soon as the pressure disappeared the index went back exactly to the same position.

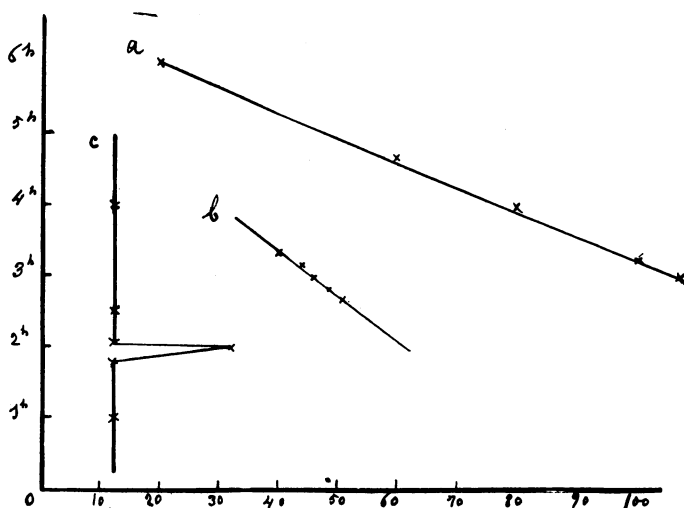


FIG. 4.

It is also required that the inner side of the capillary tube be free from dust particles and from any other imperfection changing the mobility of the index in its full run from one end to the other; it is not difficult to obtain this when using chemically clean tubes and liquids which adhere to the walls; anyway it can be tested with complete accuracy by letting the index fall along the tube a little inclined and recording its position at different intervals of time; if the friction is constant the movement will become and remain uniform. How faithfully this condition was fulfilled appears in the

diagrams Fig. 4, in which the position of the index has been plotted against the time; *a* was obtained with a drop of acetic acid, inclining the tube so as to have a force of about one hundredth of a milligram acting upon the index; the ordinates are measured in hours and the abscissae in millimeters; *b* was obtained with a drop of ether, the component of gravity acting upon it being one three-hundredth of a milligram; here the divisions of the ordinate represent minutes; *c* shows stationary oil index obtained with an arrangement like the one in Fig. 3, altho the two balloons were here very much smaller in order to secure a uniform temperature; the sudden displacement of the index was caused by applying the hand to one of the bulbs; about fifteen minutes later, the index was found again in the original position.

Coming now to the temperature, it can affect the instrument in two ways: by not being uniform, and by not remaining constant. The error arising from the first source is given by formula

$$\Delta V = S\Delta h = \frac{VP \left(\frac{T^1}{T} - 1 \right)}{P \frac{T^1}{T} + p + \frac{S}{V}}, \text{ where } T \text{ is to be measured in absolute}$$

Centigrade degrees, *P* and *p* are the pressure at *V* and *V*¹, and these two volumes are supposed to be equal. This error can be practically eliminated by arranging the two volumes *V* and *V*¹ of Fig. 2, in a series of bulbs very close together (see Plate II, A) and, if necessary, placing all the bulbs inside another one enveloping them, which would also eliminate better the effect arising from any change of pressure from the outside. Practically no great danger is to be feared from this source.

The effect of any change in temperature, supposing it to be uni-

$$\text{form, is given by the formula } \Delta V = \frac{Vh \left(\frac{T^1}{T} - 1 \right)}{\frac{T^1}{T} (P + p) + \frac{S}{V}}, \text{ where } P, p \text{ and}$$

h are to be measured in millimeters of mercury.

Comparing (*a*) the increment of volume ΔV_1 , due to Δg equal only to 10^{-6} ; (*b*) the increment ΔV_2 due to a difference of temperature of one-thousandth between the two volumes separated by the column of mercury; and (*c*) the increment of volume due to a

change of temperature of one-thousandth on both volumes, and taking as constants for the instrument

$$\begin{aligned} V^1 &= V = 10^6 \text{ cu. mm (cubic millimeters)} \\ S &= 10^4 \text{ sq. mm} \\ h &= 500 \text{ mm} \\ P &= 750 \text{ mm of Hg.} \\ p &= 250 \text{ mm of Hg.} \\ T &= 300^\circ \text{ abs. C.} \end{aligned}$$

we shall have:

$$\begin{aligned} \Delta V_1 &= 0.45 \text{ cu. mm} \\ \Delta V_2 &= 2.27 \text{ cu. mm} \\ \Delta V_3 &= 1.36 \text{ cu. mm} \end{aligned}$$

ΔV_2 , which has the greatest value, can be practically eliminated; ΔV_3 , or the increment of volume arising from a variation of temperature of one-thousandth of a degree Centigrade, is still three times greater than ΔV_1 corresponding to a variation of one-millionth in the value of g and about equal to the one caused by variation of g equal to three-millionths, corresponding to an increment of the distance from the center of the Earth, R , of about nine meters, as can be seen from the formula $\frac{g_0}{g} = \frac{(R+d)^2}{R^2} = 1 + \frac{2d}{R} + \frac{d^2}{R^2}$.

Hence, by measuring the temperature in thousandths of a degree Centigrade, heights could be measured within ten meters independently of the atmospheric pressure, and with a constant sensitiveness at any height. A spiral tube in a horizontal plane would allow a wider range for the index, or it could be used by proceeding in steps. In taking readings the instrument should be suspended.

The industrial applications of such a differential gravimeter for use as an *altimeter* will be limited by the industrial facilities in measuring temperatures. But that is not the case with very important laboratory work. Even without the proper means for it, I have kept the temperature constant within one-thousandth of a degree Centigrade during many hours, and no serious difficulties would be presented in observatories and laboratories provided with special rooms for constant temperature. Besides that, the temperature varies so slowly with the proper isolation of the instrument, that the movement of the index can be practically determined qualitatively and quantitatively within a rather great range of time during which the action of g could be tested, and its effect easily distinguished from the curve due to the temperature.

It seems that in dealing with laboratory work, the only limit to the sensitiveness of the instrument comes from the friction of the index. When using ether, which shows the greatest mobility in a

capillary tube, I found that, for a tube of one square millimeter section and a column of ether one millimeter long, the friction is smaller than one two-hundredths of a milligram. By projecting the shadow of the index upon a graduated scale the sensitiveness of the instrument could be carried well beyond $1 \cdot 10^{-7}$.

Let me now enumerate some of the main features of the complete instrument as shown in Plate II, A. The two bulbs with the terminals of the column of mercury are very much flattened so as to have a great surface, minimizing at the same time the effect that any difference of temperature between these two bulbs, which cannot be kept together, could have on the final results. The capillary tube is removable and attached to the instrument with a completely air-tight junction as shown in Fig. 5. The stop-cocks are of the form shown in Fig. 6, which prevent any air

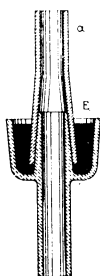


FIG. 5.

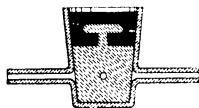


FIG. 6.

from escaping from the bulbs; when the three stop-cocks below the capillary tube are open, communication is established between the two sections of the volume V^1 (Fig. 2) and the index can be brought to any convenient position; when the central stop-cock alone is closed, differences of pressure are transmitted to the index; the closing of the other two cuts off the indicating tube from the oscillations of the mercury when moving the instrument; the other stop-cock, below at the left, is for pumping out the air until a convenient height of the column of mercury is obtained, while the last one at the right is for the introduction of mercury.

The three small cylindrical bulbs disposed horizontally at the upper part of the instrument and which can be seen in a transverse section on the left side of the plate, are intended to contain ether, thus preventing the evaporation of the index; one of these bulbs

communicates with the volume V and the other two with both regions of the volume V^1 . They have to be kept close together in order to secure a uniform temperature; when oil is employed, for work not requiring such a high degree of sensibility, these three bulbs can be eliminated.

A complete model with all these necessary details has not yet been finished; but a trial instrument was made in the Ryerson Laboratory of the University of Chicago, where Prof. A. A. Michelson, its director, gave us every facility for carrying on the preliminary experiments, and on his recommendation Dr. Ishida, with an interest and a constancy which I deeply appreciate, constructed the model which can be seen in Plate II, B. The lack of proper junctions and stop-cocks, and the excessive height of the column of mercury (determined by the distance between the lower and the upper bulbs) which, together with the vapors of ether, gave a pressure far in excess of that due to the outside atmosphere, were responsible for a continuous leaking of the instrument, which prevented quantitative measurements; but even so, the great sensitivity of the instrument was made manifest by an indirect process. When starting a movement of the instrument upwards or downwards, or when giving a new acceleration in the direction of the vertical, the column of mercury must, on account of its greater inertia, increase or decrease the pressure upon the gas, and consequently produce a sudden increment of volume. Accordingly we brought the apparatus to an elevator; as soon as we started upwards the push was so great that the index was suddenly carried out of the capillary tube; in coming down the phenomenon was repeated in the opposite direction. We then placed the instrument on a small platform which could be raised very slowly by means of a worm screw; the acceleration was so small that nobody could detect directly the rising of the instrument; nevertheless every time that I pushed the screw in one or another direction the index was displaced from twenty to thirty millimeters in the capillary tube.²

At present that is all that can be positively asserted about this differential gravimeter. As to its possible applications for the future I may point out the following:

²From this it is clear how the instrument can be employed as an *accelerimeter* by establishing a narrow communication between the two sections of volume V^1 of Fig. 2, which would destroy the permanent effect due to the change of g and at the same time would let sudden changes of g be transmitted to the index. In using the instrument as an *altimeter* the effect of sudden changes, due to a change of acceleration, could be eliminated by conical valves properly adapted between the two sections of the volume.

(1) For the geophysical observatories, in order to study variations of g in possible connection with earthquakes, or if the instrument is constructed as an accelerimeter to use it as an exceedingly sensitive seismograph in recording the vertical component.

(2) For possible laboratory work in studying the nature of g . If by means of a strong magnetic field gravitation could be changed in one ten-millionth, the instrument would detect this variation.

(3) To study the value of g on different places of the Earth with possible connection with some small changes in latitude.

(4) For comparing the mass of a ton with the mass of the Earth, or, in other words, for weighing the Earth, possibly repeating with less difficulty the experiments carried on by Cavendish in 1798.

(5) For detecting the change in the field of gravitation due to the position of the Moon on the horizon, which amounts to a little more than one ten-millionth.

(6) For measuring heights independently of the atmospheric pressure, if constancy of temperature could be secured within one-hundredth of a degree.

These are the main applications, some of which I hope to be able to test in the Observatorio del Ebro, Tortosa, Spain; meanwhile I would consider it a great reward for my work if I could contribute in any way to break down the barrier which has prevented us from entering into the dominion of the force controlling the Universe.

September, 1919.